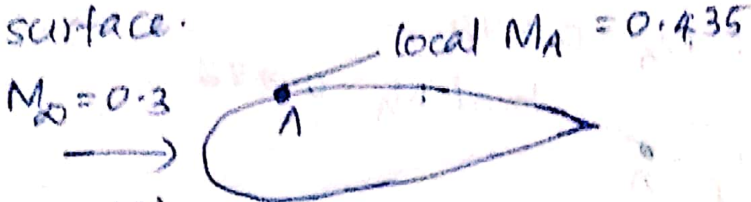
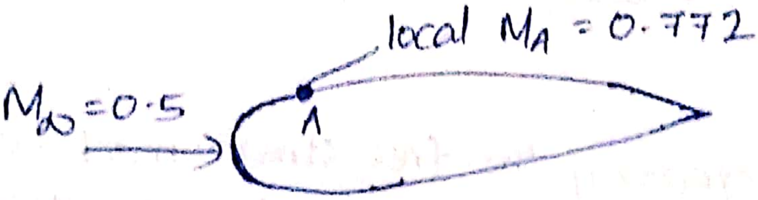
* Critical Mach number (M^*)

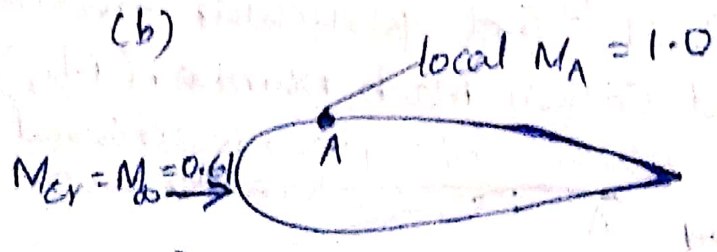
Consider an air-foil in a low speed flow with $M_\infty = 0.3$ In the expansion over the top surface of the airfoil the local flow mach number M increases. Let point A represent the location on the air-foil surface where the pressure is a minimum, hence where mach number is a maximum. When the free stream mach number (M_∞) increases, the local mach number (M_A) is also increases. The critical mach number is that free stream mach number at which sonic flow is first achieved on the air-foil surface.



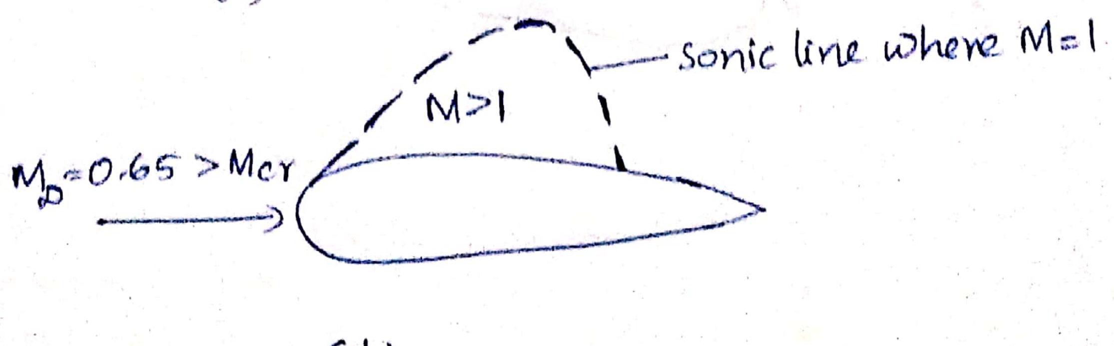
(a)



(b)



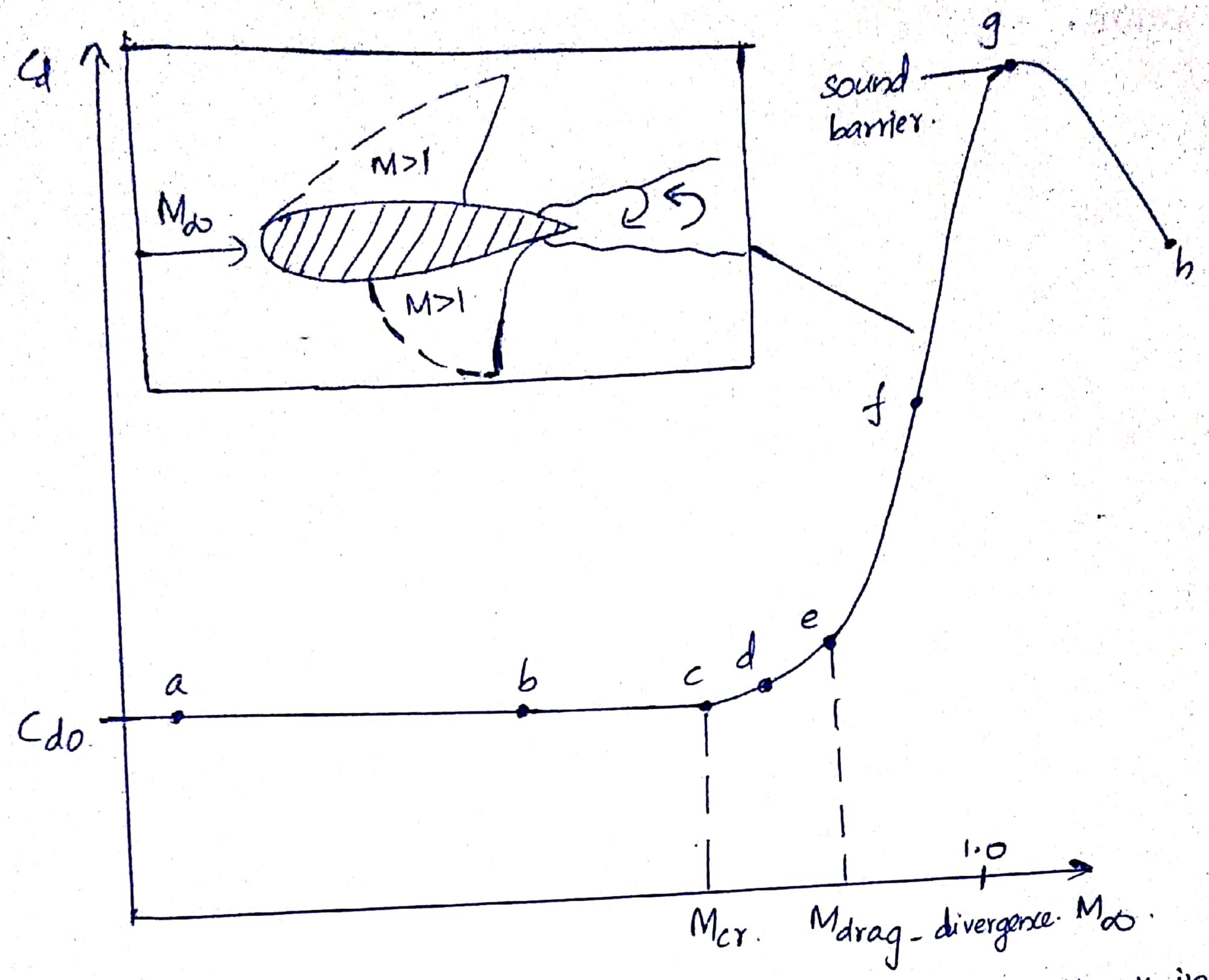
(c)



(d)

When mach number is increased above critical Mach number ($M_\infty = 0.65 > M_{cr}$) we obtain a sonic line where M=1 and it encloses a supersonic flow region where $M > 1$.

* **Drag-divergence Mach number.**



Consider an airfoil fixed at a particular angle of attack in a wind tunnel. Let C_{do} be the drag coefficient at low subsonic speed. The free stream mach number (M_∞) is increasing gradually, then C_d remains relatively constant all the way to the critical mach number. When we are increasing free stream mach number (M_∞) slightly above critical mach number (M_{cr})(point d), a finite region of supersonic flow appears on the airfoil. The mach number in this bubble of supersonic flow is only slightly above mach 1, typically 1.02 to 1.05. When free stream mach number, M_∞ is increased again the drag coefficient suddenly starts to increase (point e). The value of free stream mach number (M_∞) at which this sudden increase in drag starts is defined as the drag-divergence Mach number.

Beyond the drag divergence Mach number, the drag coefficient can become very large, typically increasing by a factor of 10 or more. This large increase in drag is associated with an extensive region of supersonic flow over the airfoil terminating in a shock wave. Corresponding to the point f on the drag curve as M_∞ approaches unity, the flow on both top and bottom surfaces can be supersonic, both terminated by shock waves.

Consider the case of a thick airfoil designed for low speed applications when M_∞ is beyond drag divergence, the local mach number can reach 1.2 or higher, as a result the terminating shock waves can be relatively strong. These shocks can cause severe flow separation downstream of the shocks with a large increase in drag.

Scientists believed that human made vehicles would never fly faster than the speed of sound till 1936. So its termed as sound barrier at point g when M=1.

Prandtl-Glauert's Equation:

$$C_p = \frac{C_{p0}}{\sqrt{1-M_\infty^2}}$$

According to Prandtl-Glauert rule

$$C_p = \frac{C_{p0}}{\sqrt{1-M_\infty^2}} \quad \text{and} \quad C_D = \frac{C_{D0}}{\sqrt{1-M_\infty^2}}$$

when $M_\infty = 1$ C_D will become infinite; so the Prandtl-Glauert rule is invalid at $M_\infty = 1$.

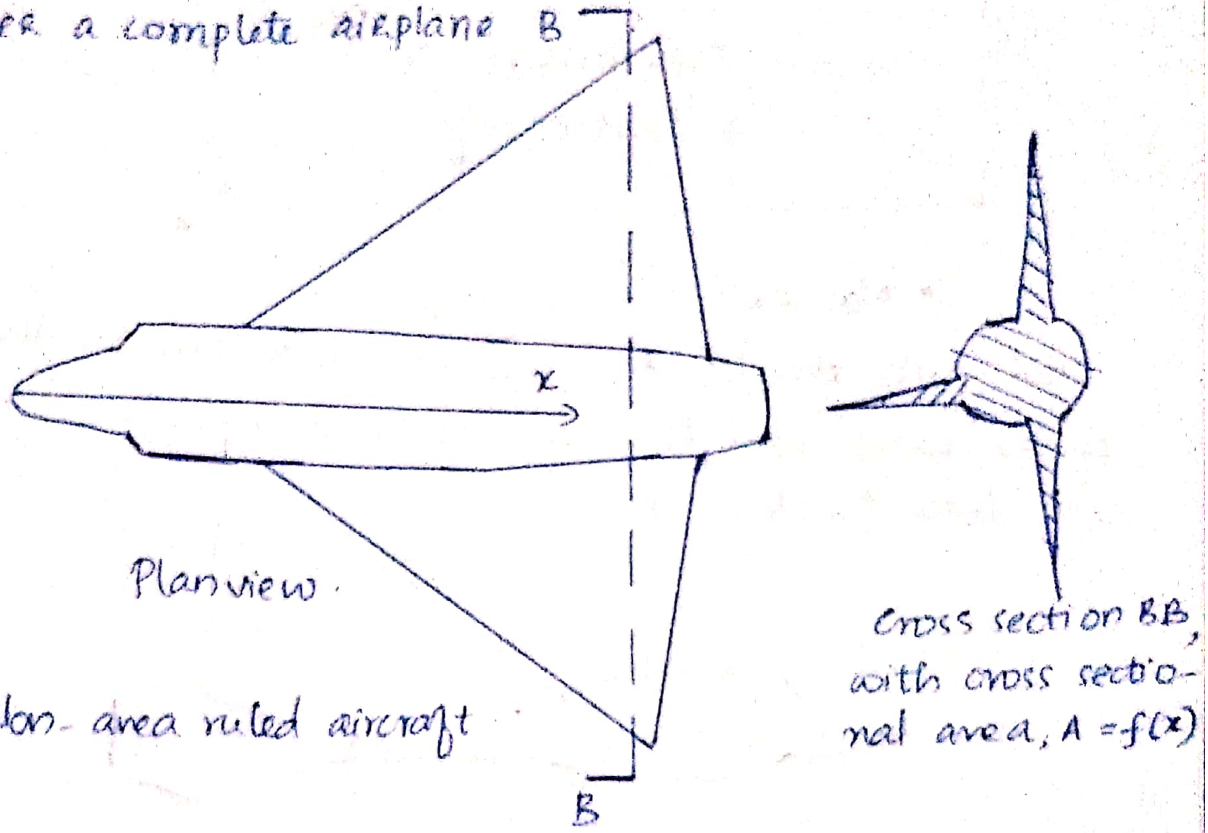
C_D peaks at or around mach number 1 and then decreases in the supersonic regime. (point gh)

* AREA RULE:

Two revolutionary concepts have developed to breakdown the sound barrier near and beyond the speed of sound.

1. The Area rule
2. Supercritical airfoil

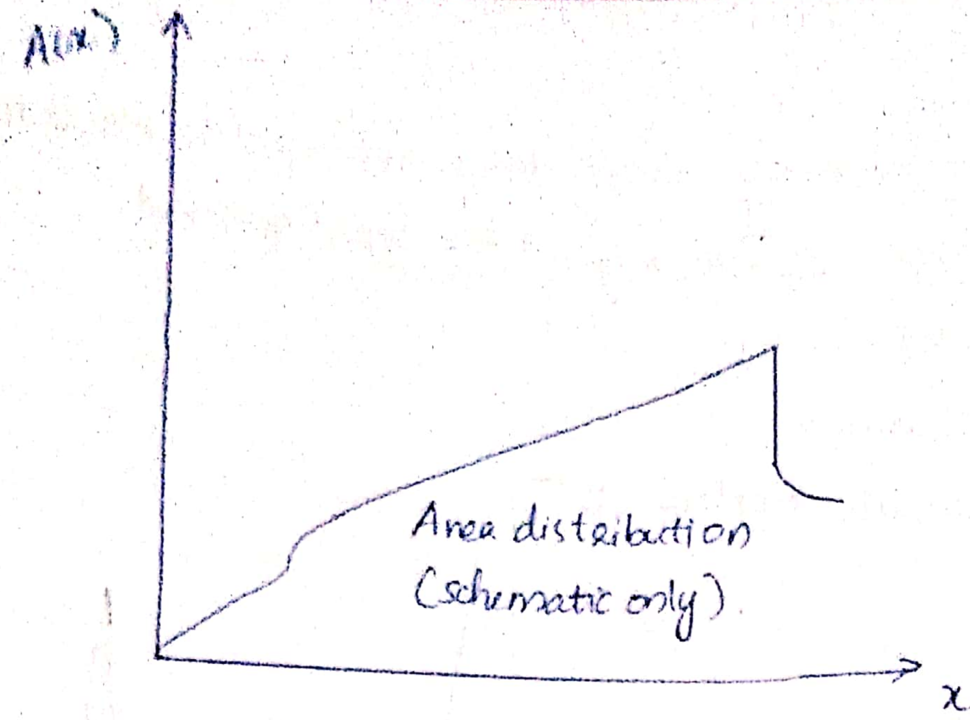
Consider a complete airplane



• Non-area ruled aircraft

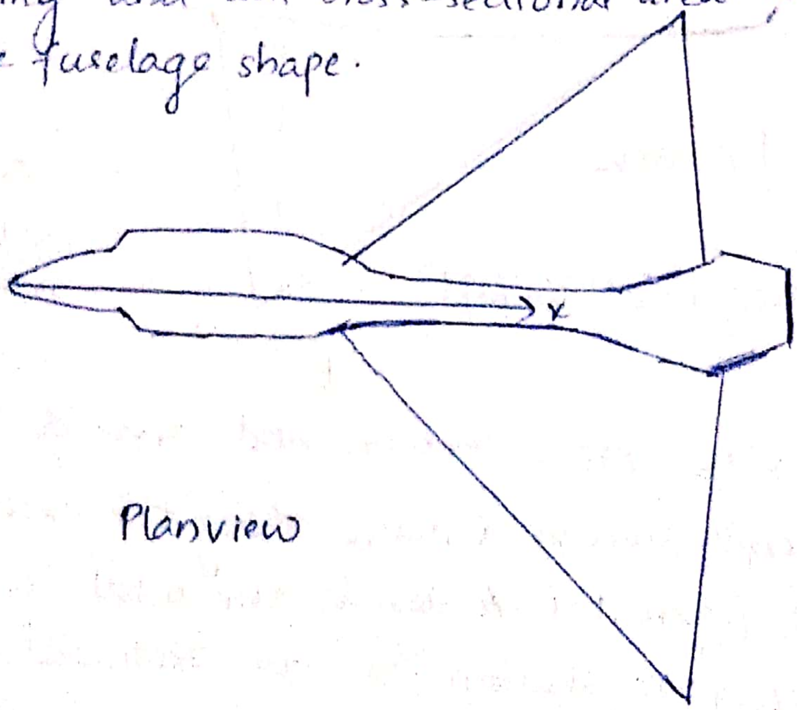
A plan view, cross section and area distribution (cross-sectional area Vs distance along the axis of aeroplane) is shown in figure. Let A denote the total cross-sectional area at any given station. The cross sectional area distribution experiences some abrupt changes along the axis, with discontinuities in both A and $\frac{dA}{dx}$ in the regions of the wing.

Richard. T. Whitcomb introduced the area rule. According to this rule, the variation of cross-sectional area of an airplane should be smooth with no discontinuities. This meant that in the region of the wings and tale, the fuselage cross-sectio-

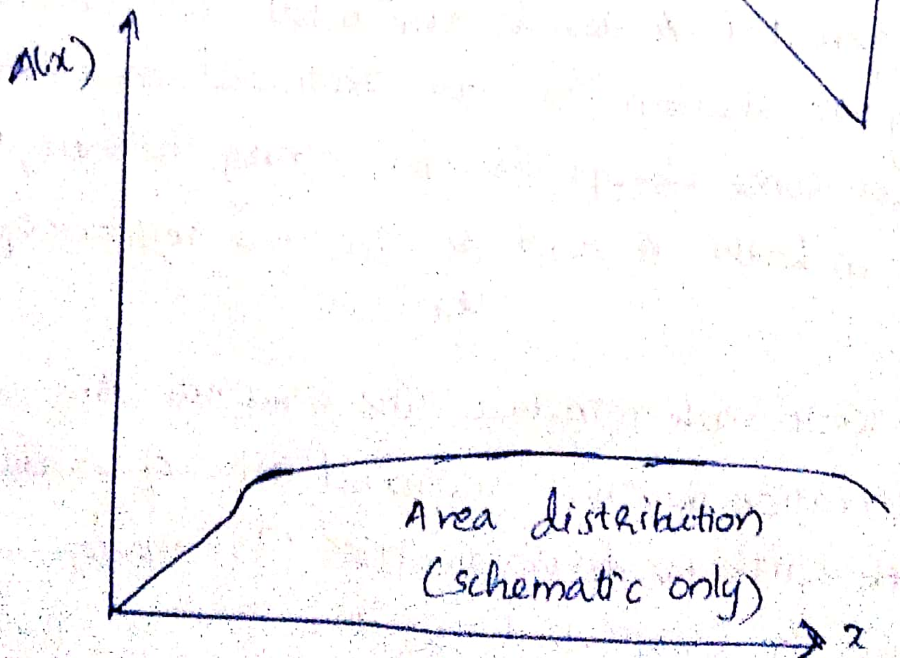


• Non-area ruled aircraft

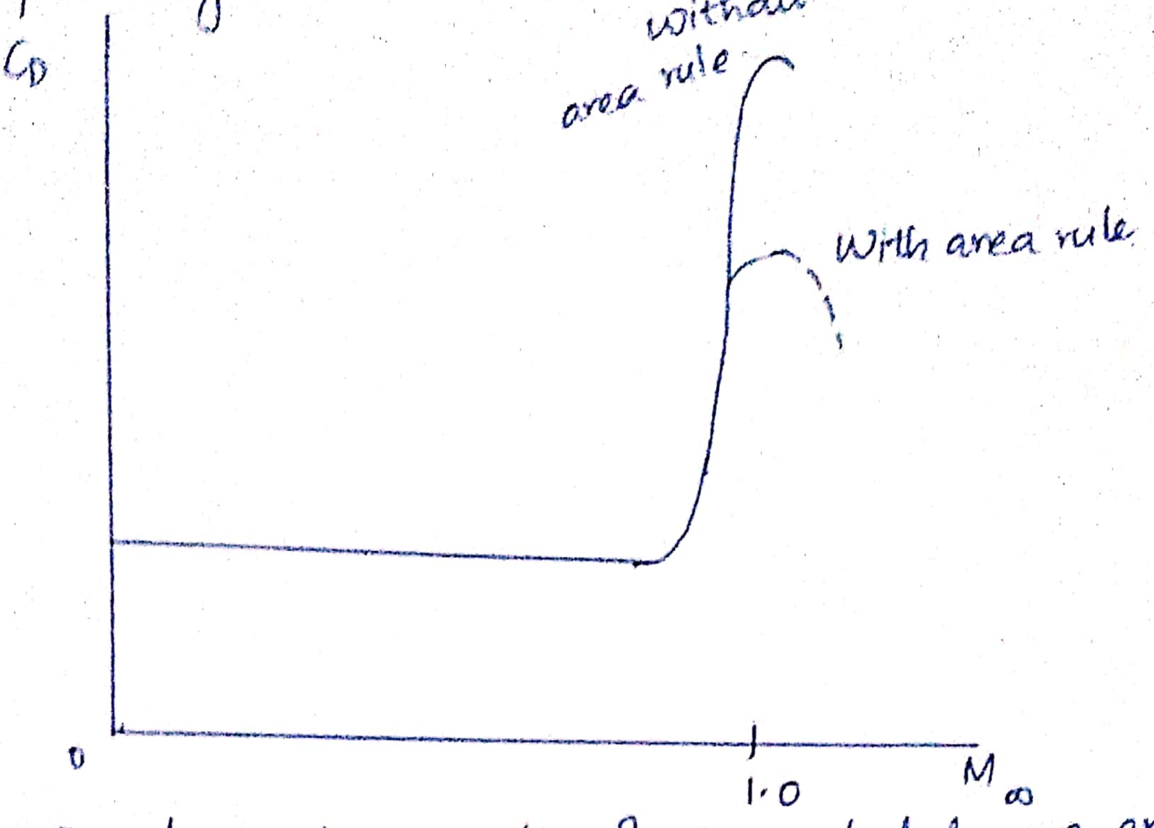
nal area should decrease to compensate for the addition of the wing and tail cross-sectional area. This led to a cock bottle fuselage shape.



Planview



The plan view and area distribution for an aircraft with a relatively smooth variation of A(x) is shown in figure. This design philosophy is called the Area rule and it successfully reduced the peak drag near mach 1.



- The drag-rise properties of area-ruled & non-area-ruled aircraft (schematic only).

The variations of C_D with M_∞ for an area ruled and non-area-ruled airplane are schematically compared in the graph. The area-rule leads to a factor of 2 reduction in the peak drag near mach 1.

* Supercritical airfoil:

* **Supercritical airfoil:**

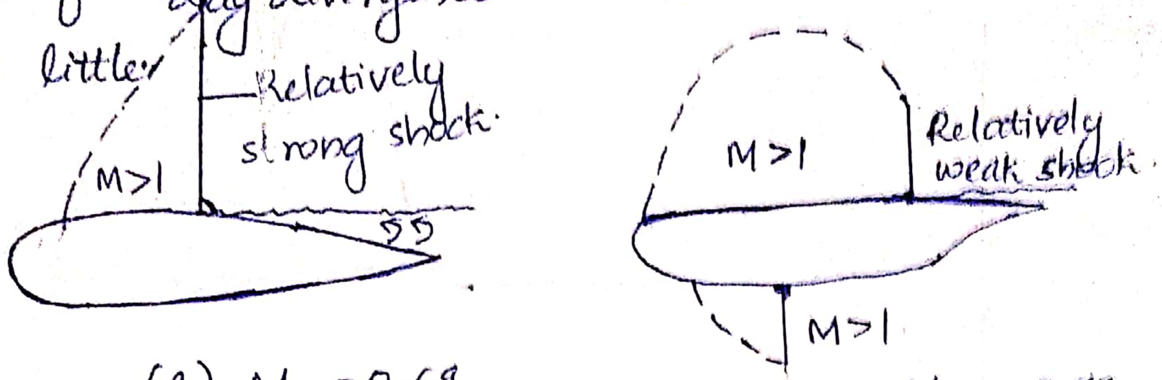
• Supercritical airfoils are specially designed profiles to increase the drag divergence mach number.

• An airfoil with a high critical mach number is required for high speed subsonic aircraft.

• When the critical Mach number increases the drag divergence mach number also increases.

• The thinner airfoils have higher value of critical mach number than the thick airfoils.

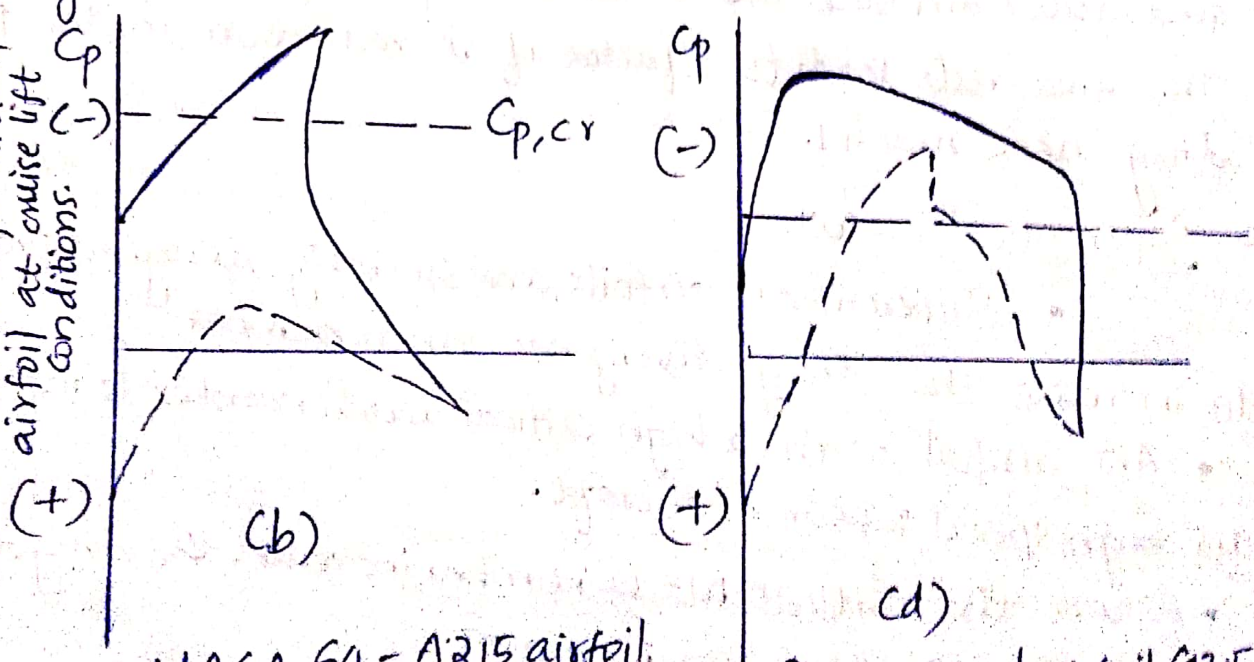
- Rather than increasing M_{cr} the Mach number increment b/w M_{cr} & $M_{drag\text{-}divergence}$ can be increased.
- This philosophy leads to the design of a new family of airfoils called <u>supercritical airfoils</u>.
- The purpose of a supercritical airfoil is to increase the value of $M_{drag\ divergence}$ although M_{cr} may change a very little.



(a) $M_\infty = 0.69$

(c) $M_\infty = 0.79$.

- The Supercritical airfoil has a relatively flat top thus encouraging a region of supersonic flow with lower local values of mach number than the NACA 64 series.
- The terminating shock is weaker and, thus creating less drag.

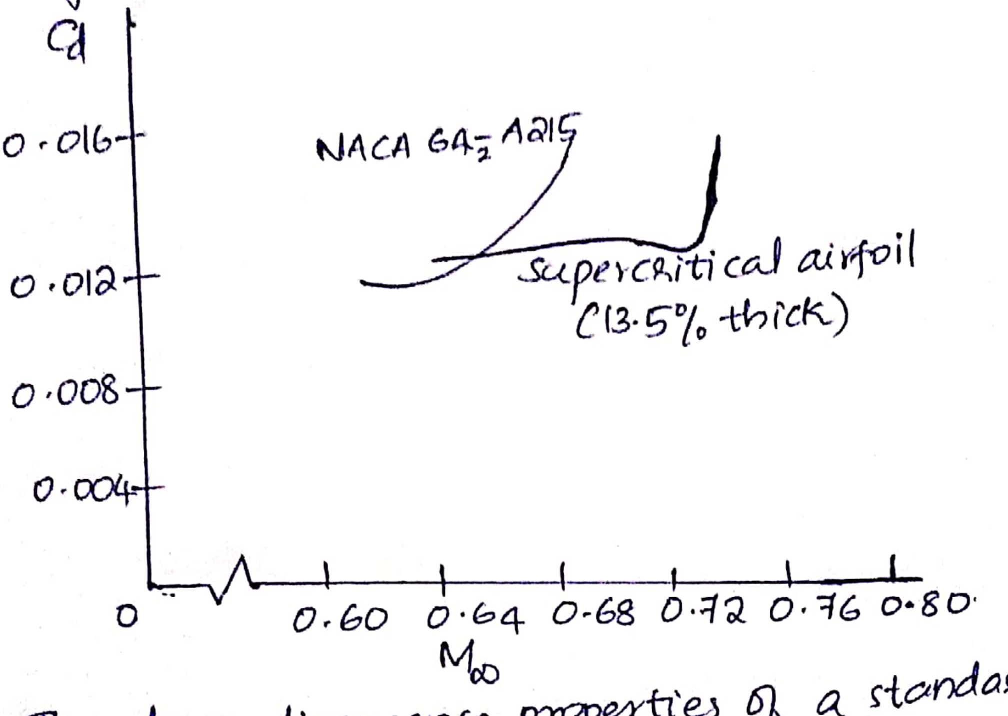


NACA 64 - A215 airfoil, $M_\infty = 0.69$

Supercritical airfoil (13.5% thick) $M_\infty = 0.79$.

- Standard NACA 64-series airfoil compared with a supercritical airfoil at cruise lift conditions.

In figure (a) and (b) for the NACA 64 series airfoil for lower free stream Mach number $M_\infty = 0.69$ than figure (c) and (d) for the supercritical airfoil at a higher free stream mach number $M_\infty = 0.79$. inspite of the fact that 64 series airfoil is at a lower M_∞, the extend of the supersonic flow reaches farther above the airfoil, the lower supersonic mach numbers are higher, and the terminating shock wave is stronger. But in the case of supercritical airfoil the local supersonic mach numbers are lower and the terminating shock wave is weaker as a result the value of drag divergence mach number will be higher for the supercritical airfoil.



• The drag-divergence properties of a standard NACA 64-series airfoil and a supercritical airfoil.

The top of the supercritical airfoil is relatively flat, the forward 60% of the airfoil has -ve camber, which lowers the lift. To compensate the lift is increased by having extreme positive camber on the rearward 30% of the airfoil. This is the reason for the cusp like shape of the bottom surface near the trailing edge.

* Rayleigh flow (Simple Heating):

The flow involving change in stagnation temperature or the stagnation enthalpy of a gas stream which flows at constant area and without frictional effects are known as Rayleigh flow. External heat exchange, combustion or moisture condensations are the examples of energy effects.

For a flow of gas through a constant area duct without friction the momentum equation may be written as:

$$\boxed{P + \rho V^2 = \frac{F}{A} = \text{constant}} \longrightarrow (1)$$

where F is impulse function.

By Continuity equation:

$$\dot{m} = \rho A V$$

$$\boxed{\frac{\dot{m}}{A} = \rho V = G = \text{constant}} \longrightarrow (2)$$

$$\left[F = PA + \rho A V^2 = PA\left[1 + \gamma M^2\right] \right.$$

G is mass velocity.

from eqn (2) : $V = \dfrac{G}{\rho} \longrightarrow (3)$

sub eqn (3) in eqn (1) :

$$\boxed{P + \rho \frac{G^2}{\rho^2} = \frac{F}{A}}$$

$$\therefore \boxed{P + \frac{G^2}{\rho} = \frac{F}{A}} \longrightarrow (4)$$

⇒ for constant value of G & $\frac{F}{A}$, equation $P + \frac{G^2}{\rho} = \frac{F}{A}$ defines a unique relation b/w P & ρ is called Rayleigh line.

⇒ Since both the enthalpy & entropy S are functions of pressure and density.

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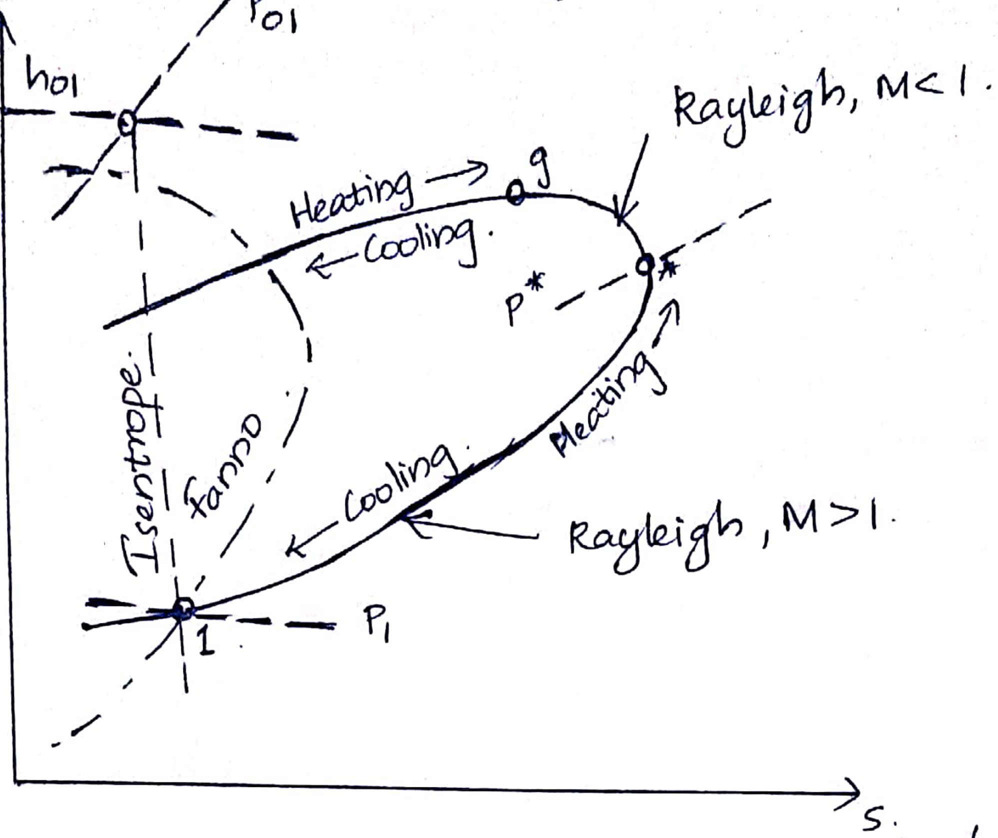
Eqn ④ is used for representing Rayleigh line on h-s diagram.



Most of the fluids in practical use have rayleigh curves of the general form is shaon in figure. The portion of the Rayleigh curve above the point of maximum entropy corresponds to subsonic flow and the portion belao the point of maximum entropy corresponds to supersonic flow. The process simple heating is thermodynamically reversible. During heat addition entropy increases and during heat rejection entropy decreases.

At subsonic speed: (i) By heating Mach number increases.
(ii) By cooling Mach number decreases.

At supersonic speed: (i) By heating Mach number decreases.
(ii) By cooling Mach number increases.

The heat addition will make the mach number in the duct unity. For heat addition at either subsonic or supersonic speeds the amount of heat input cannot be greater than that for which the leaving mach number is unity. If the heat addition is too large the flow will be choked, ie., the initial mach number will be reduced to a magnitude which is consistent with specified amount of heat input.

* Swept Wing:

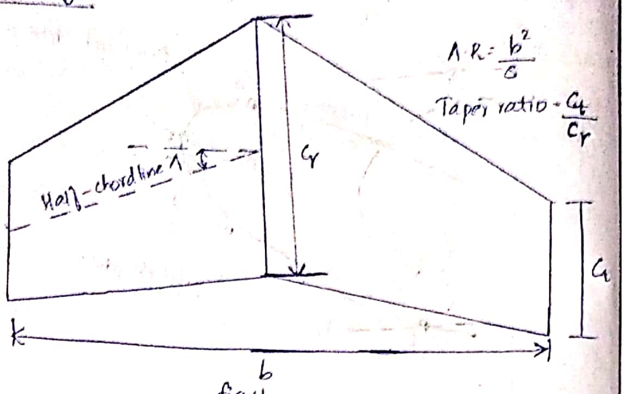


fig :1

In the transonic regime, there is a large ↑se in drag asscociated with the formation of the shock waves and with boundary layer interaction. These 2 effects are known as wave drag and boundary layer shock drag respectively.

When an aircraft enters the transonic speed just below the speed of the sound, an effect known as wave drag starts, near the speed of sound oblique shock wave is generated. It requires energy to form. It reduces the airplane's power. Various design features have introduced to overcome or reduce the transonic drag rise.

(i) They may increase the value Mcr

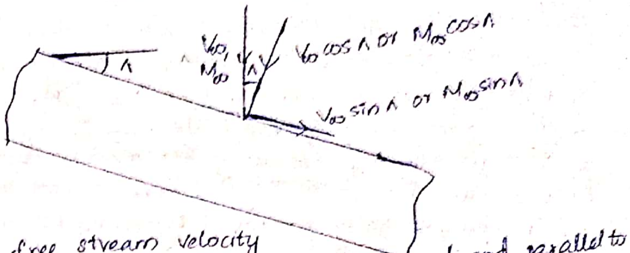
(ii) various design features concerning the wing section, wing planform, fuselage and tail plane of an aircraft.

→ Wing planform:

There are 2 features of wing planform which have a significant effect on critical Mach number:

(i) Sweep back (ii) Aspect ratio.

Consider an infinite wing swept back at an angle Λ as shown in fig 1.



The free stream velocity vector can be resolved into components normal and parallel to the leading edge. And the mach number associated with these components are $M_\infty \cos\Lambda$ and $M_\infty \sin\Lambda$ respectively. The component of the flow ‖el to the L.E does not affect the aerodynamic properties of the wing and the pressure distribution will depend only on the value of $M_\infty \cos\Lambda$.

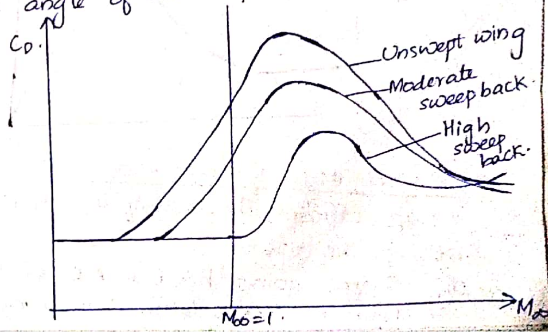
If the critical mach number for the same wing when unswept is denoted by M^*, then the critical condition for swept wing will arise when $M_\infty \cos\Lambda = M^*$

For the swept wing:

$$M_\infty = M_{critical}$$

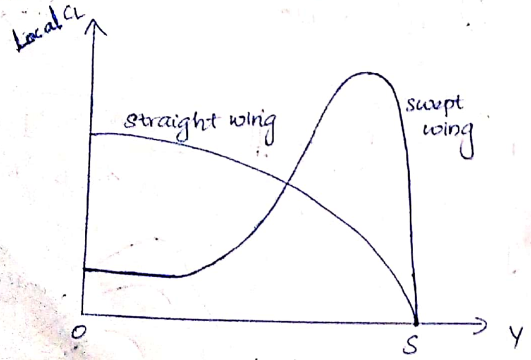
$$\therefore M_{critical} = \frac{M^*}{\cos\Lambda} \Rightarrow \boxed{M_{critical} = M^* \sec\Lambda}$$

Thus, Increasing the angle of the sweep will ↑se the Mcritical indefinitely.



Sweep back not only ↑ses the M_{cr} but also reduces the rate at which C_D rises in the transonic region. However when supersonic flow is achieved there is not the same reduction in the C_D with ↑sing Mach number as occurs with the unswept wing and at higher Mach numbers with a given angle of sweep, the value of C_D is slightly above that for the unswept wing. The mach number at which this first occurs ↑ses with angle of sweep so that this effect can be progressively postponed by using higher sweep back angles.

Reduction in aspect ratio gives a rise in M_{cr}. It leads to the reduction of transonic drag. This reduction more than outweighs any ↑se in induced drag, so that low aspect ratio is a desirable feature at high subsonic and transonic speeds. for swept wing drag will ↑ses at low speed.



* **Disadvantages of swept wing:**

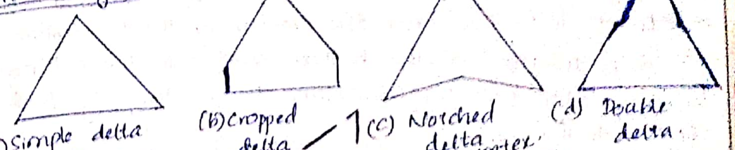
(i) Tip stalling – The stall of the swept wing tends to occur first at the tips.

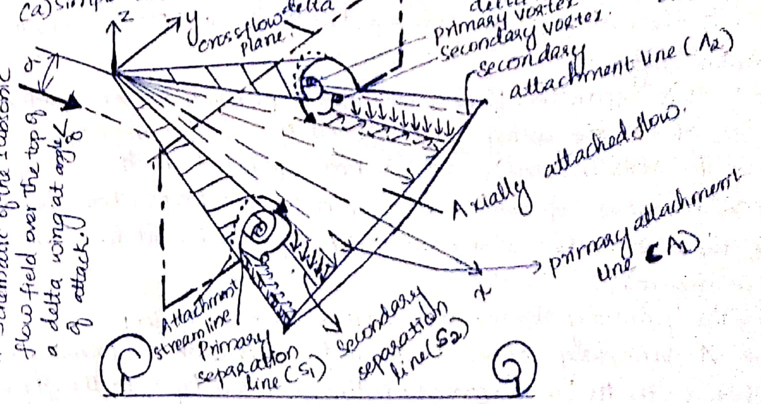
(ii) Swept wing has low A.R, so the ~~the~~ D_i is more at high incidence.

(iii) Lateral control may be poor.

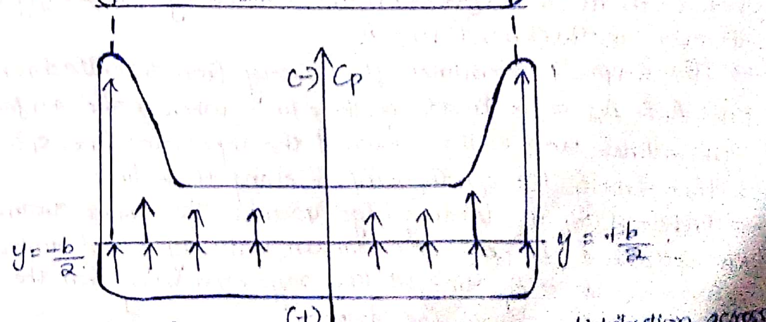
* **Note:** To operate at High subsonic or transonic speeds it should have low thickness, low A.R, sweep back and low wing loading (W/S).

* **Delta Wing:**





* Schematic of the subsonic flow field over the top of a delta wing at angle of attack.



* Schematic of the spanwise pressure coefficient distribution across delta wing.

→ A special case of swept wings is those a/c with a triangular planform - called delta wing.

→ The dominant aspect of the subsonic flow pattern over the top of a delta wing at angle of attack are the 2 vortex patterns that occurs in the vicinity of the highly swept leading edges.

→ These vortex patterns are created by the following mechanism.

→ The pressure on the bottom surface of the wing at the angle of attack is higher than the pressure on the top surface.

→ Thus the flow on the bottom surface in the vicinity of the leading edge tries to curl around the leading edge from the bottom to the top.

→ If the leading edge is sharp, the flow will separate along its entire length.

→ This separated flow curls into a primary vortex which exists above the wing just inboard of each leading edge.

→ The stream surface which has separated at the leading edge (the primary separation line S_1 in figure) loops above the wing & then reattaches along the primary attachment line (line A_1 in figure).

→ The primary vortex is contained within the loop.

→ A secondary vortex is formed underneath the primary vortex, with its own separation line, denoted by S_2 in the figure & its own reattachment line A_2.

→ The surface streamlines flow away from the attachment lines A_1 & A_2 on both sides of these lines, whereas the surface streamlines tend to flow toward the separation lines S_1 & S_2 & then simply lift off the surface along these lines.

→ Inboard of the leading edge vortices, the surface streamlines are attached & flow downstream virtually is undisturbed along a series of straight line rays emanating from the vertex of the triangular shape.

→ The surface pressure on the top surface of the delta wing is reduced near the leading edge & is higher & reasonably constant over the middle of the wing.

→ The spanwise variation of pressure over the bottom surface is constant & higher than the freestream pressure (a +ve C_p).

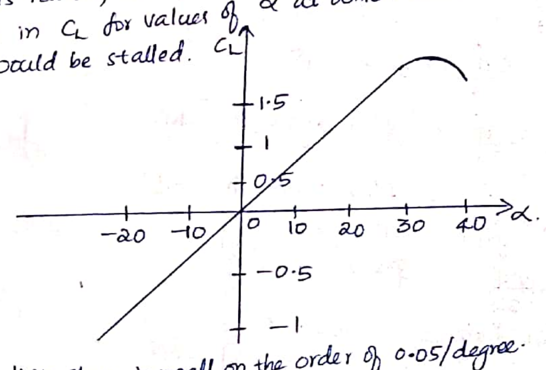
→ Over the top surface, the spanwise variation in the midsection of the wing is constant & lower than the freestream pressure (a −ve C_p).

→ Near the leading edges, the static pressure drops considerably (the values of C_p becomes more −ve).

→ The leading edge vortices are creating a strong "suction" on the top surface near the leading edges.

→ The vertical arrows are indicate the effect on the spanwise lift distribution; the upward direction of these arrows as well as their relative length show the local contribution of each section of the wing to the normal force distribution.

→ The suction effect of the leading edge vortices enhances the lift; for this reason, the lift coefficient curve for a delta wing exhibits an ↑se in C_L for values of α at which conventional wing planforms would be stalled.



(i) The lift slope is small, on the order of 0.05/degree.

(ii) The stalling angle of attack 35°; The net result is a reasonable value of $C_{L_{max}}$ on the order of 1.3.

* Linearized Supersonic flow:

→ The linearized perturbation velocity potential equation holds for both subsonic & supersonic flow.

$$\boxed{(1-M_\infty^2)\frac{\partial^2\hat\phi}{\partial x^2}+\frac{\partial^2\hat\phi}{\partial y^2}=0} \longrightarrow (1)$$

If $1-M_\infty^2>0 \Rightarrow$ subsonic flow

If $1-M_\infty^2<0 \Rightarrow$ supersonic flow.

→ Let $\sqrt{M_\infty^2-1}=\lambda$; Eqn (1) becomes $\boxed{\lambda^2\frac{\partial^2\hat\phi}{\partial x^2}+\frac{\partial^2\hat\phi}{\partial y^2}=0}\longrightarrow (A)$

→ A solution to this equation is the functional relation:

$$\boxed{\hat\phi = f(x-\lambda y)} \longrightarrow (2)$$

$$\frac{\partial\hat\phi}{\partial x} = f'(x-\lambda y)\frac{\partial}{\partial x}(x-\lambda y)$$

$$\frac{\partial\hat\phi}{\partial x}=f'(x-\lambda y)\longrightarrow(3)$$

Eqn (3) can be written as:

$$\frac{\partial\hat\phi}{\partial x}=f'\longrightarrow(4)$$

$$\frac{\partial\hat\phi}{\partial y}=f'(x-\lambda y)\frac{\partial}{\partial y}(x-\lambda y)$$

$$= f'(x-\lambda y)\times-\lambda \longrightarrow (5)$$

Eqn (5) can be written as:

$$\frac{\partial\hat\phi}{\partial y}=-f'\lambda\longrightarrow(6)$$

Differentiate eqn (3) w.r.t x:

$$\frac{\partial^2\hat\phi}{\partial x^2}=f''(x-\lambda y)\times 1\longrightarrow(7)$$

Eqn (7) can be written as:

$$\frac{\partial^2\hat\phi}{\partial x^2}=f''\longrightarrow(8)$$

Differentiate Eqn (5) w.r.t y:

$$\frac{\partial^2\hat\phi}{\partial y^2} = -\lambda f''(x-\lambda y)\times-\lambda$$

$$= \lambda^2 f''(x-\lambda y)\longrightarrow(9)$$

Eqn (9) can be written as:

$$\frac{\partial^2\hat\phi}{\partial y^2}=\lambda^2 f''\longrightarrow(10)$$

Substitute eqn (8) & eqn (10) in eqn (A):

$$\boxed{\lambda^2 f''-\lambda^2 f''=0}$$

Hence, eqn (A) is indeed a solution of eqn (1).

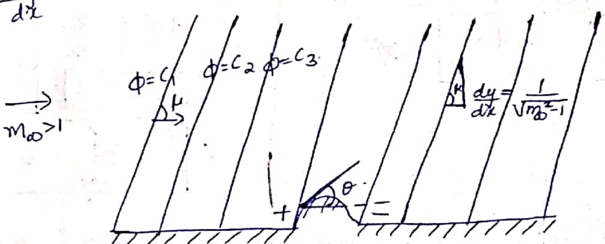
⇒ $\hat\phi$ is constant along lines of $x-\lambda y=$ constant $\longrightarrow (11)$

⇒ Differentiate eqn (11) w.r.t x =

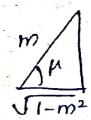
$$1-\lambda\frac{dy}{dx}=0$$

$$\boxed{\frac{dy}{dx}=\frac{1}{\lambda}=\frac{1}{\sqrt{M_\infty^2-1}}}\longrightarrow(12)$$

$\frac{dy}{dx}$ is the slope of the line.



$$\boxed{\tan\mu=\frac{1}{\sqrt{M_\infty^2-1}}}\longrightarrow(13)$$



$\mu=\sin^{-1}\left(\frac{1}{m}\right)$

$$\hat u = \frac{\partial\hat\phi}{\partial x}$$

$$\boxed{\hat u = f'}\longrightarrow(14)$$

$\hat{v} = \frac{\partial \hat{\phi}}{\partial y}$

$\boxed{\hat{v} = -\lambda f'} \longrightarrow (15)$; $\boxed{f' = -\frac{\hat{v}}{\lambda}} \longrightarrow (16)$

Equate eqn (14) & (16)

$\boxed{\hat{u} = -\frac{\hat{v}}{\lambda}} \longrightarrow (17)$

→ From linearized velocity potential eqn:

$\boxed{\frac{\partial \hat{\phi}}{\partial y} = V_\infty \tan\theta} \longrightarrow (18)$

∴ $\boxed{\hat{v} = \frac{\partial \hat{\phi}}{\partial y} = V_\infty \tan\theta} \longrightarrow (19)$

→ for small perturbation θ is small

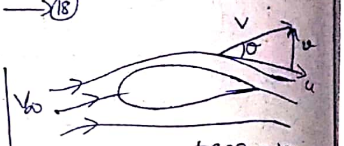
∴ $\tan\theta \approx \theta$

→ Eqn (19) becomes

$\boxed{\hat{v} = V_\infty \theta} \longrightarrow (20)$

→ Substitute the eqn (20) in eqn (17):

∴ $\boxed{\hat{u} = -\frac{V_\infty \theta}{\lambda}} \longrightarrow (21)$



$\tan\theta = \frac{v}{u}$

where $u = \hat{u} + V_\infty$

$\hat{u}$ = perturbation Velocity

∴ $\tan\theta = \frac{\hat{v}}{V_\infty + \hat{u}}$; $\hat{u} << V_\infty$

$\tan\theta = \frac{\hat{v}}{V_\infty}$

$\boxed{v' = V_\infty \tan\theta}$ & $\boxed{\hat{v} = \frac{\partial \hat{\phi}}{\partial y}}$

∴ $\boxed{\hat{v} = \frac{\partial \hat{\phi}}{\partial y} = V_\infty \tan\theta}$

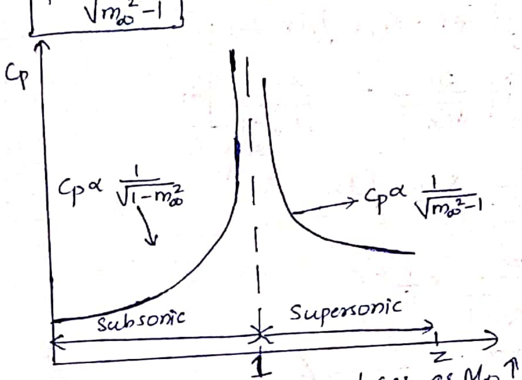
The linearized pressure coefficient is given by eqn:

$\boxed{C_p = -\frac{2\hat{u}}{V_\infty}} \longrightarrow (22)$

→ Substitute the value of $\hat{u}$ in eqn (22)

$C_p = \frac{-2 \times -V_\infty \theta}{V_\infty \lambda} = \frac{2\theta}{\lambda}$; where $\lambda = \sqrt{M_\infty^2 - 1}$

∴ $\boxed{C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}}} \longrightarrow (23)$

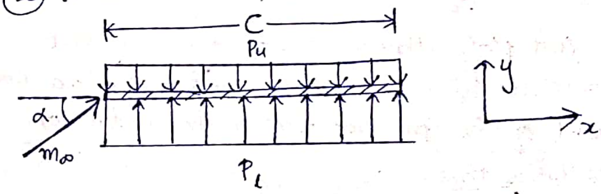


→ For supersonic flow, C_p ↓ses as M_∞ ↑ses.

→ For subsonic flow; C_p ↑ses as M_∞ ↑ses.

→ when $C_p \to \infty$; $M_\infty = 1$

→ Eqn (23) is not valid in transonic range around mach 1.



$C_{p,l} = \frac{2\alpha}{\sqrt{M_\infty^2 - 1}} \longrightarrow (24)$

$C_{p,u} = \frac{-2\alpha}{\sqrt{M_\infty^2 - 1}} \longrightarrow (25)$

$C_{p,l}$ is constant over the lower surface and $C_{p,u}$ is constant over the upper surface.

→ The normal force coefficient for the flat plate is:

$$C_n = \frac{1}{c}\int_0^c [C_{p,l} - C_{p,u}]\,dx \longrightarrow (26)$$

$$= \frac{1}{c}\int_0^c \left[\frac{2\alpha}{\sqrt{M_\infty^2-1}} - \frac{-2\alpha}{\sqrt{M_\infty^2-1}}\right]dx$$

$$= \frac{4\alpha}{\sqrt{M_\infty^2-1}} \times \frac{1}{c}\int_0^c dx$$

$$= \frac{4\alpha}{\sqrt{M_\infty^2-1}} \times \frac{1}{c} \times c$$

$$\therefore C_n = \frac{4\alpha}{\sqrt{M_\infty^2-1}} \longrightarrow (27)$$

→ The axial force coefficient:

$$C_a = \frac{1}{c}\int_{LE}^{TE} [C_{p,u} - C_{p,l}]\,dy$$

→ For a flat plate, thickness is zero; $dy = 0$; So $C_a = 0$.

→ The pressure act normal to the surface and hence there is no component of the pressure force in the x-direction.

We know that:

$$\boxed{\begin{aligned} L &= N\cos\alpha - A\sin\alpha \\ D &= N\sin\alpha + A\cos\alpha \end{aligned}}$$

where; N = Normal force

A = Axial force

$$\therefore C_l = C_n\cos\alpha - C_a\sin\alpha \longrightarrow (28)$$

$$C_d = C_n\sin\alpha + C_a\cos\alpha \longrightarrow (29)$$

α is small; $\therefore \cos\alpha \approx 1$; $C_a = 0$; So eqn (28) & (29) becomes:
$\sin\alpha \approx \alpha$

$$C_l = C_n\cos\alpha = C_n$$

$$\therefore \boxed{C_l = \frac{4\alpha}{\sqrt{M_\infty^2-1}}} \longrightarrow (30)$$

$$C_d = C_n\,\alpha$$

$$\therefore \boxed{C_d = \frac{4\alpha^2}{\sqrt{M_\infty^2-1}}} \longrightarrow (31)$$

→ The pitching moment coefficient about the leading edge.

$$C_m = \frac{1}{c}\int_0^c (C_{p,l} - C_{p,u})\left(\frac{x}{c}\right)dx$$

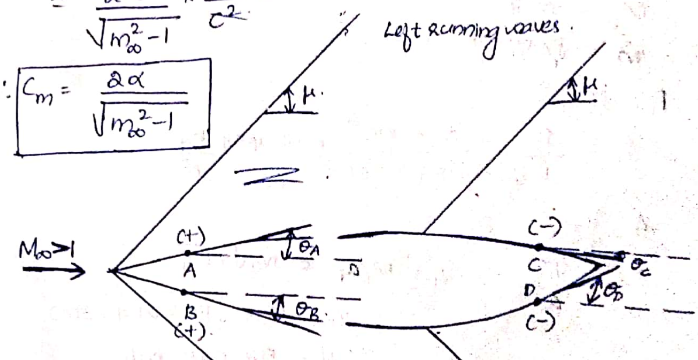
$$= \frac{1}{c^2}\int_0^c \left[\frac{2\alpha}{\sqrt{M_\infty^2-1}} + \frac{2\alpha}{\sqrt{M_\infty^2-1}}\right] x\,dx$$

$$= \frac{4\alpha}{\sqrt{M_\infty^2-1}} \times \frac{1}{c^2}\int_0^c x\,dx$$

$$= \frac{\cancel{4}2\alpha}{\sqrt{M_\infty^2-1}} \times \frac{1}{c^2}\left[\frac{x^2}{2}\right]_0^c$$

$$= \frac{2\alpha}{\sqrt{M_\infty^2-1}} \times \frac{1}{c^2} \times c^2$$

$$\therefore \boxed{C_m = \frac{2\alpha}{\sqrt{M_\infty^2-1}}}$$



Scanned with CamScanner

1. Air flows through an adiabatic pipe at a rate of 8kg/s
The pressure, temperature and Mach number of air at the
entrance of the pipe are 0.3 MPa, 300K and 0.15 respectively.
The coefficient of friction for the pipe is constant and its
value is 0.004. The mach number of the pipe at the exit
is 0.7. Find (a) stagnation pressure losses,
(b) Length of the pipe.
(c) diameter of the pipe.
(d) pressure, temperature and density of air at the exit of the pipe?

Ans: Given data:

$\dot{m} = 8\frac{kg}{s}$

$P_1 = 0.3 MPa$

$T_1 = 300K$

$M_1 = 0.15$

$\bar{f} = 0.004$

$M_2 = 0.7$

(b) $\dot{m} = \rho_1 A V_1 = 8$

$P_1 = \rho_1 R T_1$

$\therefore \rho_1 = \frac{P_1}{RT_1} = \frac{0.3 \times 10^6}{287 \times 300} = 3.484 \frac{Kg}{m^3}$

$A = \frac{\pi}{4} d^2$

$M_1 = \frac{V_1}{a_1} \Rightarrow \therefore V_1 = M_1 a_1 = M_1 \sqrt{\gamma R T_1}$

$V_1 = 0.15 \sqrt{1.4 \times 287 \times 300}$

$\therefore V_1 = 52.078 \ m/s$

$\therefore \dot{m} = \rho_1 A V_1$

$A = \frac{\dot{m}}{\rho_1 V_1} = \frac{8}{3.484 \times 52.078} = 0.04409 m^2$

$\therefore A = \frac{\pi}{4} D^2$

$\therefore D = \left(\frac{4A}{\pi}\right)^{1/2} = \left(\frac{4 \times 0.04409}{\pi}\right)^{1/2} = 0.2369 m$

$\frac{4\bar{f}L_{max}}{D} = 28.3545$ (from one-dimensional flow with friction ($\gamma = 1.4$) for $M_1 = 0.15$)

$\therefore L_{max} = \frac{28.3545 \times 0.2369}{4 \times 0.004} = 419.8238 m$

$\therefore$ Length of the pipe $= 419.8238 m$

(c) Diameter of the pipe, $D = 0.2369 m$

(d) For $M_1 = 0.15$

$\frac{P_1}{P^*} = 7.3191$

For $M_2 = 0.7$

$\frac{P_2}{P^*} = 1.49345$

$\therefore \frac{\frac{P_2}{P^*}}{\frac{P_1}{P^*}} = \frac{P_2}{P_1} = \frac{1.49345}{7.3191} = 0.20404$

$\therefore P_2 = 0.20404 P_1 = 0.20404 \times 0.3 \times 10^6$

$\therefore P_2 = 61214.493 Pa$

$\therefore P_2 = 0.061214 MPa$

for $M_1 = 0.15$

$\frac{T_1}{T^*} = 1.1946$

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For $M_2 = 0.7$

$$\frac{T_2}{T^*} = 1.09290.$$

$$\therefore \frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{1.09290}{1.1946} = 0.91486.$$

$$\therefore T_2 = 0.91486 \times T_1 = 0.91486 \times 300.$$

$$\therefore T_2 = 274.460\,K.$$

$$\therefore \rho_2 = \frac{P_2}{RT_2} = \frac{0.061214 \times 10^6}{287 \times 274.460} = 0.7771\ \frac{kg}{m^3}$$

~~for values~~
From isentropic table

for $M_1 = 0.15$

$$\frac{P_1}{P_{01}} = 0.9844 \Rightarrow P_{01} = \frac{0.3 \times 10^6}{0.9844} = 0.30475\,MPa.$$

for $M_2 = 0.7$

$$\frac{P_2}{P_{02}} = 0.7209. \Rightarrow P_{02} = \frac{0.061214 \times 10^6}{0.7209} = 0.08491\,MPa$$

(a) $\therefore P_{02} - P_{01} = -0.21984$ MPa. ('-ve' sign indicates that Pressure decreases). (Stagnation pressure losses)

2. In a fanno flow the initial Mach number is 0.2 and the static pressure & static temperature of 300K and 200KPa respectively. If the duct is 0.1m diameter & the frictional factor is 0.004. find the mach number, static pressure, static temperature and the total pressure after the flow has travelled 45 metre of the duct.

Ans: Given data:

$M_1 = 0.2$.

$T_1 = 300K$

$P_1 = 200KPa = 200 \times 10^3$ Pa.

$f = 0.004$.

$L_{max} = 45m$.

$D = 0.1m$.

For $M_1 = 0.2$

$$\frac{T_1}{T^*} = 1.19048, \quad \frac{P_1}{P^*} = 5.45545, \quad \frac{P_{01}}{P_0^*} = 2.96352.$$

$$\frac{4fL_{max}}{D} = \frac{4 \times 0.004 \times 45}{0.1} = 7.2.$$

From table 7.2 is b/w 7.6876 & 6.3572.

$$\frac{7.6876 - 7.2}{7.6876 - 6.3572} = \frac{0.26 - M_2}{0.26 - 0.28}$$

$$\therefore \boxed{M_2 = 0.2673}$$

For $M_2 = 0.2673$

from table 0.2673 is b/w 0.26 & 0.28.

for $\frac{T_2}{T^*}$:

$$\frac{0.28 - 0.2673}{0.28 - 0.26} = \frac{1.18147 - T_2/T^*}{1.18147 - 1.18399}$$

$$\therefore \frac{T_2}{T^*} = 1.18307.$$

for $\frac{P_2}{P^*}$:

$$\frac{0.28 - .2673}{.28 - 0.26} = \frac{3.88199 - P_2/P^*}{3.88199 - 4.18506}.$$

$$\therefore P_2/P^* = 4.07443$$

for $\frac{P_{02}}{P_0^*}$:

$$\frac{0.28 - 0.2673}{.28 - 0.26} = \frac{2.16555 - P_{02}/P_0^*}{2.16555 - 2.31729}.$$

$$\therefore \frac{P_{02}}{P_0^*} = 2.26190$$

$$\frac{T_1}{T_2} \Rightarrow \frac{T_1/T^*}{T_2/T^*} = \frac{1.19048}{1.18307} \Rightarrow \boxed{T_2 = 298.132K}$$

$$\frac{P_1}{P_2} = \frac{\frac{P_1}{P^*}}{\frac{P_2}{P^*}} = \frac{5.45545}{4.07443} \Rightarrow \boxed{\begin{aligned} P_2 &= 149.370\,KPa \\ &= 149.370 \times 10^3\,Pa. \end{aligned}}$$

$$\frac{\frac{P_{01}}{P_0^*}}{\frac{P_{02}}{P_0^*}} = \frac{P_{01}}{P_{02}} = \frac{2.96352}{2.26190} \Rightarrow \frac{P_{01}}{P_{02}} = 1.31019.$$

we know, $\frac{P_{01}}{P_1} = \left(1 + \frac{r-1}{2} M_1^2\right)^{\frac{r}{r-1}}$

$$\therefore P_{01} = \left[1 + \frac{1.4-1}{2} 0.2^2\right]^{\frac{1.4}{1.4-1}} \times 200 \times 10^3$$

$$= 205656.224\ Pa$$

$$\boxed{\therefore P_{01} = 205.656\,KPa}$$

Illy $P_{02} = \left[1 + \frac{r-1}{2} M_2^2\right]^{\frac{r}{r-1}} \times P_2$.

$$= \left[1 + \frac{1.4-1}{2} 0.2673^2\right]^{\frac{1.4}{1.4-1}} \times 149.370.$$

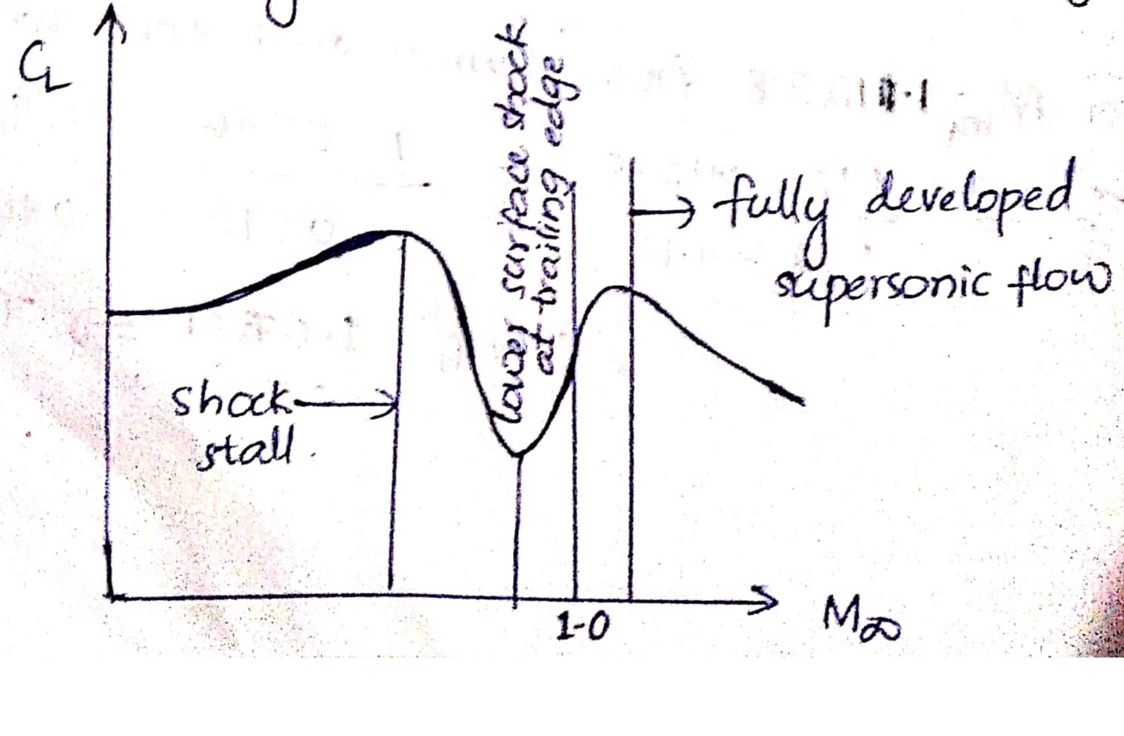
$$\therefore \boxed{P_{02} = 156.975\,KPa. = 156975.0649\,Pa}.$$

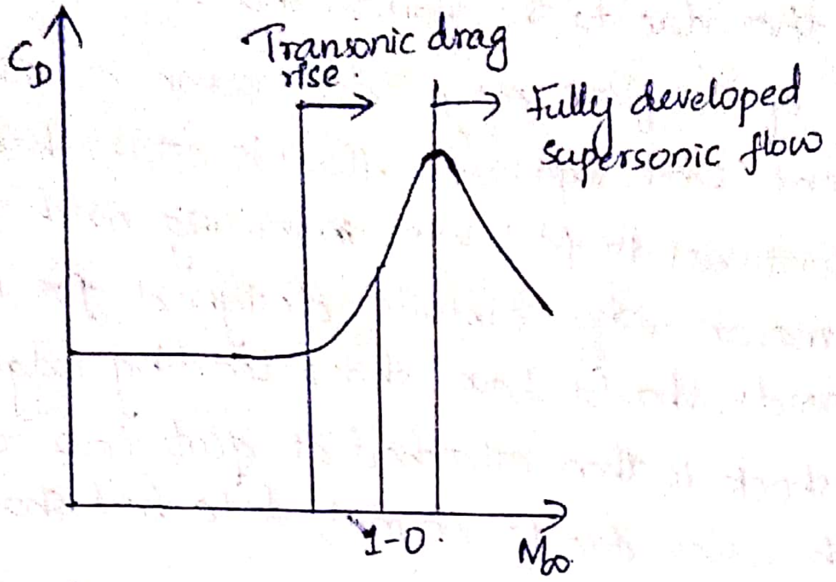
2

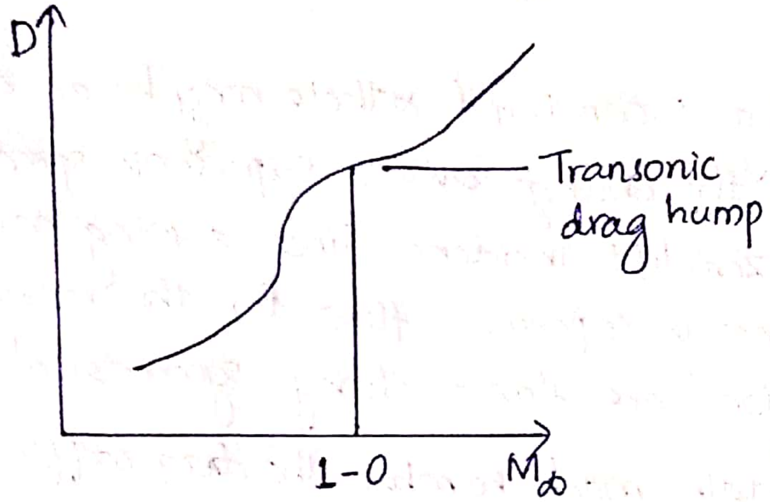
* Shock stall:

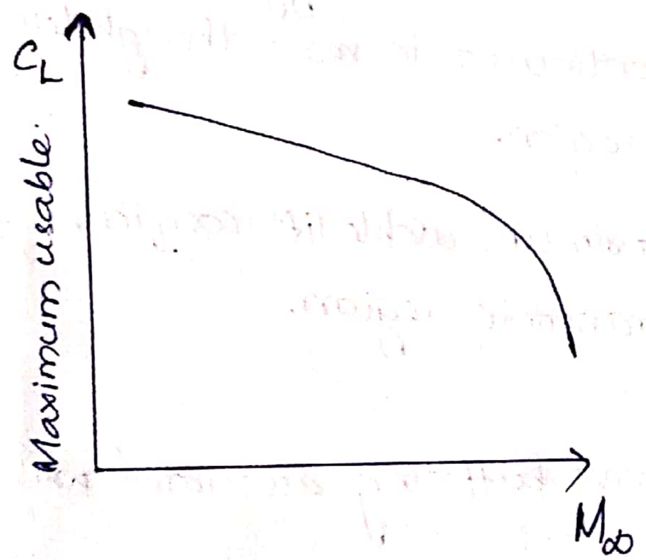
When the free stream mach number (M_∞) increases above the critical mach number, the upper surface shock wave moves backwards and it strengthens..

It becomes strong enough to cause the flow to separate. When this happens the lift coefficient begins to fall and the drag to rise rapidly. This phenomenon is known as the shock stall. It differs from the conventional stall in that it may occur at low incidence. At higher incidence it will occur at lower mach number. The associated increase in drag is known as the transonic drag rise









As the upper surface shock moves backwards with increase in M_∞ the region of shock induced separation is reduced and once the lower surface shock is established at the trailing edge some measure of recovery of lift may occur. The drag rise due to the oblique shock's at the trailing edge will also generally be less than that due to the normal shocks. In addition the reattachment of the flow may cause some reduction in the drag coefficient. Once supersonic flow is established the drag coefficient continues to fall with increasing mach number. This is one reason why airfoils designed for use at supersonic speeds should have sharp leading edges. The leading edge shock is then attached at quite low supersonic speeds and the losses due to normal, detached shocks are avoided.

If a wing is cambered there may be an additional loss of lift at the change over to supersonic speeds due to the change in zero lift incidence. Since a wing has a positive zero lift incidence in supersonic flow. For this reason the supersonic wing sections are almost always symmetrical.

3rd graph → Beyond a certain mach number the drag coefficient falls with increasing M_∞; this is not to suggest that the drag force also falls. The drag continues to increase though less rapidly than in the transonic region.

4th graph → The value of maximum usable lift coefficient falls off very rapidly in the transonic region.

(Buffeting).

Because of the intensive buffeting associated with the shock stall.